

FREE FLOW OF A GRANULAR MATERIAL FROM AN APERTURE IN THE PRESENCE OF A GAS COUNTERFLOW

G. G. Kuvshinov

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1. INTRODUCTION

The problem of free flow of a granular material from a single aperture is the classical hourglass problem but there are no composite physical pictures of the mechanism of the process. Even less study has been made of the free flow of a dispersed material from a single aperture under conditions that are typical of the operation of a system of return flow of dispersed material in equipment with stationary, moving, fluidized, or circulating layers, as well as dispersed material hoppers and feeders, where the free flow of dispersed material from an aperture takes place against a gas counterflow. Besides the free flow velocity, the critical gas velocity at which particles cease to flow freely is an important practical parameter in this case.

A review of publications on this subject is contained in [1]. The largest number of those publications deal with gravity flow of a dispersed material from an aperture when no gas flow exists. Experiments showed that the flow velocity in this case is almost independent of the height of the granular material layer [2], provided that the height of the layer above the aperture does not exceed the aperture diameter. Kennerman et al. [2] also showed that the nature of the free flow does not depend on the presence and location of immobile elements in the layer above the aperture, if they are at a height greater than the aperture diameter. These results are very important since they indicate that the flow velocity does not depend on the nature of the particle motion above the aperture but rather is determined by the emergence of particles from the dense layer into free space.

The empirical relations proposed in various publications [3-8] for calculating the mass discharge rate of a dispersed material through an aperture in the absence of a gas flow can be reduced to

$$j_m = K\Pi\rho_d S_0 (gd_0)^{1/2}, \quad (1.1)$$

where k is a constant dimensionless coefficient, Π is a correction factor that depends on the aperture/particle diameter ratio, $\rho_d = \rho_s(1 - \varepsilon_0)$ is the bulk density of the dispersed material, ρ_s is the apparent particle density, ε_0 is the porosity of the immobile layer, S_0 and d_0 are the aperture area and diameter, and g is the free fall acceleration.

Table 1 shows the principal differences between the relations given in the literature for calculating the free flow velocity of truly free-flowing dispersed material. The square brackets contain the values introduced into the original formulas so that could be reduced to the form (1.1). The main difference consists in the specific way that the influence of particle size has been taken into account. Comparative calculations indicate that all the relations are in fairly good agreement, the exception being the formula in line 3, which was evidently given with an error in [5].

Very little experimentation has been done on the free flow of particles in the presence of a gas counterflow [4, 5, 9, 10]. It has been determined [4, 5, 10] that as the gas velocity increases the flow velocity of the dispersed material decreases and at the critical gas velocity W_{ac} the particle motion ceases altogether or becomes pulsational with a very small flow of material through the aperture. We know of no relations for calculating the flow velocity of a dispersed material from an aperture in the presence a gas counterflow as well as the values of W_{ac} .

Theoretical attempts to describe the process of free flow of a dispersed material from an aperture have been undertaken only for the case when there is no gas flow in the aperture. Studies on this subject cannot be considered to have been successful since the relations obtained in them contain empirical coefficients and the initial propositions are wrong in a number of cases.

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TABLE 1

K	Π	Note	Reference
0,55	1	—	[3]
$\frac{6,67 \cdot 0,403 [4]}{[\pi]} \eta = 0.87-0.99$	$d_0^{0.1} (1 - 2,48 \frac{d_a}{d_0} \Gamma)$	$\eta = 0.24-0.29$	[4]
$\frac{8,41 \eta [4]}{[\pi g^{1/2}]} = 0.82-0.99$ (g, m/sec ²)	$(1 - 2,48 \frac{d_a}{d_0} \Gamma) / d_0$	$\eta = 0.24-0.29$	[5]
$\frac{5730 \cdot [4]}{[\pi g^{1/2}]^2 5/2} = 0,686$ (g, cm/min ²)	$(1 - 1,8 \frac{d_a}{d_0} + 6,4 (\frac{d_a}{d_0})^2)$		[6]
$\frac{4(2)^{1/2}}{15} \lambda / [(1 - \epsilon_0)] =$ $= 0.51-0.64$	$(1 - \frac{d_a}{d_0})^{5/2}$	$\epsilon_0 = 0,4$ $\lambda = 0.82-1.02$	[7]
$0,65(1,6)^{1/2} = 0,82$	$(1 - 1,25 \frac{d_a}{d_0})^{5/2}$		[8]

In particular, the derivation of equations in [11] for the discharge rate of granular material was based on the assumption of planar packing of particles, which should not occur near the aperture; moreover, the relations obtained do not make allowance for the influence of particle size.

When deriving the relations for the discharge rate of particles Zenkov [12] assumed that the cross-sectional area and flow density are constant near the aperture for a variable particle velocity, but this is contrary to the law of conservation of mass.

Linchevskii postulates in [3] that a dynamic dome exists above the aperture, particles beneath the dome move in the free fall mode, i.e., with acceleration, and at the same time the density of the flow beneath the dome is constant and is equal to the density of the stationary layer. Clearly, this is also impossible if the law of conservation of mass is observed.

The hypothesis that is most promising for the further development of the theory of free flow of a dispersed material from an aperture is that a dynamic unloading dome exists above the aperture. This well explains why the velocity of free flow is independent of the height of the layer and the distinctive features of a granular medium above the aperture, provided that the height of the free granular layer above the aperture is greater than the aperture diameter.

2. ELEMENTARY THEORY OF FREE FLOW OF A GRANULAR MATERIAL FROM AN APERTURE

The proposed theory is based on the aforementioned hypothesis that a dynamic dome exists above the aperture. The dynamic dome hypothesis, as mentioned in [3], was first advanced by Pokrovskii and Aref'ev. The concept of a dynamic dome is arbitrary to some degree. Particles are not tightly bound either beneath or above the dome. Since they are mobile, the distance between them should be slightly greater than in an immobile layer. The parts of the flow above and beneath the dome differ in that moving particles above the dome interact with each other. Particles moving in an ensemble reach the maximum possible velocity on the surface of the dome. Further acceleration of the interacting particles in the constricting flow causes the flow to jam and break. The velocity of free flow of the granular material is determined by the emergence of particles from the dynamic dome into free space, where the particles move in the free fall mode, without interacting with each other. To solve the problem of free flow of a dispersed material within the framework of the given model, we must consider the emergence of particles from a dynamic dome formed above an aperture.

The position of particles in the dome, the action of the forces, and the displacement of particles over the dome are depicted schematically in Fig. 1a. The dome is assumed to be spherical. Particles in the dome are in continuous motion, with the particles moving away and the dome being renewed continuously. Only part of the array of particles lying at the base of the dome on the grid are in a slightly different state. These particles come out of contact with the immobile grid substantially more quickly than do the other particles and as a result the lower array of particles of the dome remains on the grid even when they only touch the grid. Because the particles of the lower array are bound to the grid the aperture is partially covered, as it were, by particles of that array. Bearing the above in mind, we can assume this dome has a diameter that is smaller than the aperture diameter d_a by a value almost equal to the particle diameter d_s , i.e., the dome diameter $d_b = d_a - d_s$.

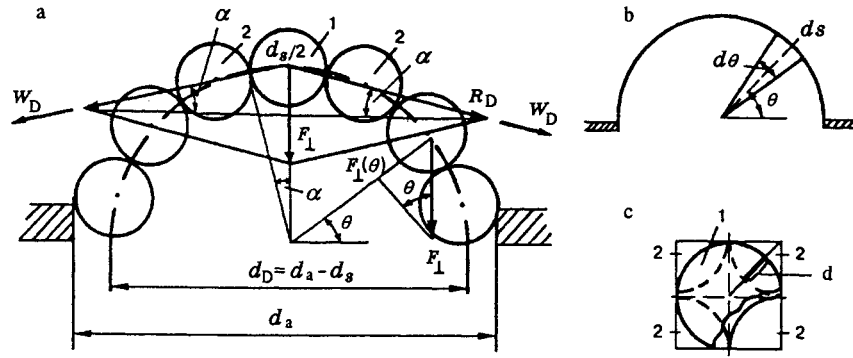


Fig. 1

Let us consider the mechanism by which a particle emerges from the dome. Suppose that this is particle 1 in Fig. 1a. In the radial direction of interest to us the particle moves in the dome under the effect of the radial component $F_{\perp}(\theta)$ of the external forces, which depends on the polar angle θ (Fig. 1b). This force is the projection of the sum of the external forces, i.e., the gravity force $F_{1\perp}$ and the friction force $F_{s\perp}$ associated with the particle, onto the radial direction. During motion the particle under consideration acts on each of the n neighboring particles, hindering their emergence (particles 2 in Fig. 1a), with a force

$$R_D = F_{\perp}(\theta)/(n \sin \alpha). \quad (2.1)$$

In this case particles 2 move on the surface of the dome with a velocity W_D . The balance of forces acting on those particles can be written as

$$R_D - R_W = m_s a_s, \quad (2.2)$$

where R_W is the resistance force due to the collision of particle 2 moving with velocity W_D in the dome with surrounding particles and $a_s = dW_D/d\tau$ is the acceleration of particle 2. That force would be the same as if a particle flux of density $\rho_s(1 - \varepsilon)$ with velocity W_D (ε is the porosity of the dome) were to impinge on a stationary particle of cross section $\pi d_s^2/4$. The force R_W is equal to the change in the momentum of the particles in the flux as a result of collisions with the given particle in a unit of time. Assuming the collisions to be elastic, we obtain

$$R_W = \rho_s(1 - \varepsilon)W_D^2 \pi d_s^2/2. \quad (2.3)$$

As follows from Fig. 1a,

$$\sin \alpha = d_s/(d_a - d_s). \quad (2.4)$$

Taking (2.1)-(2.4) into account, on the basis of (2.2) we write

$$\frac{dW_D}{d\tau} = \frac{6F_{\perp}(\theta)d_D}{\pi d_s^4 \rho_s n} - \frac{3(1 - \varepsilon)}{d_s} W_D^2, \quad (2.5)$$

whereupon, by integrating over time from 0 to τ and over velocity from 0 to W_D we determine the time τ in which particle 2 acquires a velocity W_D ,

$$\tau = \int_0^{W_D} \frac{dW_D}{A^2 - B^2 W_D^2} = \frac{1}{2AB} \ln \frac{A + B W_D}{A - B W_D}, \quad (2.6)$$

where

$$A^2 = \frac{6F_{\perp}(\theta)d_D}{\pi d_s^4 \rho_s n}, \quad B^2 = \frac{3(1-\varepsilon)}{d_s}. \quad (2.7)$$

From (2.6) we find

$$W_D = \frac{A e^{2AB\tau} - 1}{B e^{2AB\tau} + 1}. \quad (2.8)$$

Cubic packing is the most stable packing of particles in a single layer [13]. Accordingly, we assume that a particle leaving the dome a particle is in such a packing and thus acts on four neighboring particles 2, as shown in Fig. 1c. In that case we must assume that

$$n = 4, \quad \delta = d_s(2 - 2^{1/2})/2, \quad (2.9)$$

where δ is the distance by which the particle 2 must be displaced under the effect of a particle 1 leaving the dome, as is seen from Fig. 1c. On the other hand, $\delta = \int_0^{\tau_m} W_D d\tau$ (τ_m is the time taken by a particle to emerge from the dome). With allowance for (2.8), therefore, we have

$$\delta = (1/B^2) \ln \operatorname{ch}(AB\tau_m). \quad (2.10)$$

Solving (2.10) for τ_m , we obtain

$$\tau_m = (1/AB) \ln(e^{\delta B^2} + (e^{2\delta B^2} - 1)^{1/2}). \quad (2.11)$$

Taking (2.7) and (2.9) into account, we recast (2.1) in the form

$$\tau_m = (\pi d_s^5 \rho_s n / (18 d_c (1-\varepsilon) F_{\perp}(\theta)))^{1/2} \ln Z, \quad (2.12)$$

where

$$Z = e^{\delta B^2} + (e^{2\delta B^2} - 1)^{1/2}. \quad (2.13)$$

The quantity τ_m characterizes the time in which one particle emerges from the dome area $\pi d_s^2 / (4(1-\varepsilon))$, per particle in the dome. The number of particles from an elementary annular element $d s$ of the dome (Fig. 1c) per unit of time is

$$dN = 4(1-\varepsilon) d s / (\pi d_s^2 \tau_m). \quad (2.14)$$

The mass flow rate of particles from an elementary annular portion with allowance for (2.9) is $d j_m = \rho_s \pi d_s^3 / 6 d N$. Consequently, when we take (2.12) and (2.14) into account and make the substitution $d s = (\pi d_c^2 / 2) \cos \theta \cdot d\theta$, we find

$$d j_m = \frac{(1-\varepsilon)^{3/2} d_d^{5/2}}{d_s^{3/2}} \left(\frac{\pi \rho_s F_{\perp}(\theta)}{2} \right)^{1/2} \ln^{-1} Z \cos \theta \cdot d\theta. \quad (2.15)$$

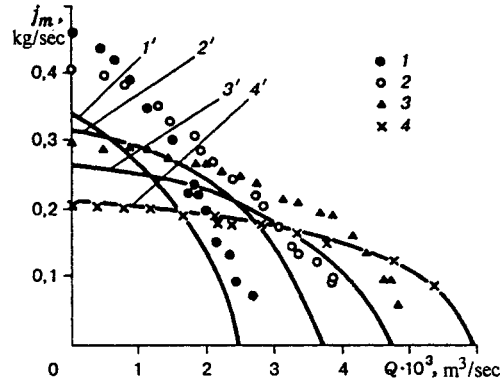


Fig. 2

To calculate the total flow rate of granular material through the aperture we must integrate (2.15) over θ from 0 to $\pi/2$. As before, we must solve the problem of determining the radial component of the external body forces $F_{1\perp}$. Obviously, for free gravity flow without gas filtration we have

$$F_{1\perp}(\theta) = \frac{\pi d_s^3}{6} \rho_s g \sin \theta. \quad (2.16)$$

The resistance force that acts on the particle and is due to gas filtration can be determined by using the Ergun equation for the pressure drop during gas filtration through a layer of stationary granular material [14]:

$$P_e = 150 \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{\rho_g \nu_g U_c}{d_s^2} + 1,75 \frac{(1-\varepsilon)}{\varepsilon^3} \frac{\rho_g U_d^2}{d_s}. \quad (2.17)$$

Here ρ_g and ν_g are the density and kinematic viscosity of the gas, P_e is pressure drop per unit segment, and U_D is the gas filtration velocity per unit dome surface.

Bearing in mind that $U_D = W_a/2$ (by virtue of conservation of gas flow rate) the gas velocity W_a at the base of the dome (velocity in the aperture) and the gas filtration rate U_D on the dome surface are inversely proportional to the areas of the dome base and surface) and the force acting on one particle from the gas is $F_g = P_e/n_1$, where the number of particles per unit volume is $n_1 = 6(1-\varepsilon)/(\pi d_s^3)$, we obtain

$$F_g = 12,5\pi \frac{1-\varepsilon}{\varepsilon^3} d_s \rho_g \nu_g W_a + 7,29 \cdot 10^{-2} \frac{\pi d_s^2 \rho_g W_a^2}{\varepsilon^3}. \quad (2.18)$$

Assuming that the direction of gas motion for particles at the dome surface coincides with aperture axis, the radial component of the friction force, like the gravity force, is written as

$$F_{g\perp} = F_g \sin \theta. \quad (2.19)$$

Now, taking (2.16) and (2.19) into account and integrating (2.15) from 0 to $\pi/2$, we have a relation for calculating the mass flow rate of a dispersed material per unit time through the aperture,

$$j_m = K \rho_d S_a \left(\frac{(\pi d_s^3 \rho_s g - 6 F_g) d_a}{\pi d_s^3 \rho_s} \right)^{1/2} \left(1 - \frac{d_s}{d_a} \right)^{5/2}, \quad (2.20)$$

where

$$K = 4((1-\varepsilon)/3)^{3/2} / (1-\varepsilon_0) / \ln Z. \quad (2.21)$$

Equation (2.20) is a generalized relation for the velocity of the free flow of a granular monodisperse material through a round aperture by gravity with no gas counterflow. This relation is not strict. Moreover, it must be noted that even though

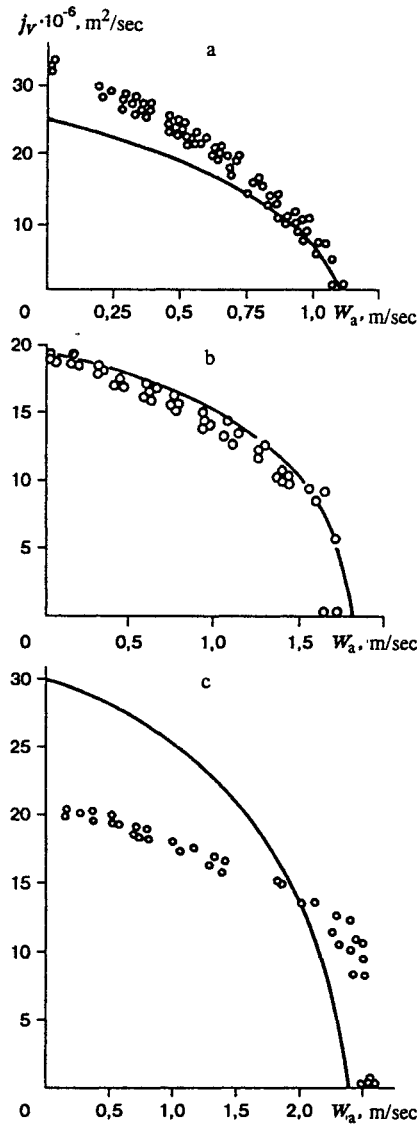


Fig. 3

it was obtained within the framework of certain assumptions about the mechanism by which particles emerge from the dome, this relation does not contain undetermined coefficients.

Within the scope of ideas about the mechanism of free flow of a dispersed material from an aperture the critical gas velocity, at which the free flow of solid particles from the aperture ceases, is found from (2.20) for the condition $j_m = 0$,

$$W_{ad} = \frac{4Ar \nu_g / d_s}{\frac{150(1-\epsilon)}{\epsilon^3} + \left(\left(\frac{150(1-\epsilon)}{\epsilon^3} \right)^2 + 4Ar \frac{1,75}{\epsilon^3} \right)^{1/2}} \quad (2.22)$$

($Ar = d_s^3 \rho_s g / (\rho_g \nu_g^2)$ is the Archimedes number).

The relations obtained do not take into account a number of factors, which are very difficult to characterize quantitatively: particle shape, moisture content, presence of electrostatic forces, polydispersity, and other factors that produce cohesive forces between particles or, conversely, increase their mobility. These factors are generally included by choosing the appropriate proportionality factor K .

3. COMPARISON OF THE THEORY WITH EXPERIMENTAL DATA

Gravity Flow of Dispersed Material from an Aperture. Numerous studies have been done on this subject. It is desirable, therefore, to consider the primary experimental data. It is more convenient to compare the theoretical relations found here with the relations given in the literature and supported by experiment that are recommended for the free flow of easily flowing materials from cylindrical apertures when there is no effect of a gas flow.

Equation (2.20) has the form

$$j_m = K \rho_d S_a (d_a g)^{1/2} (1 - d_s/d_a)^{5/2} \quad (3.1)$$

for the case considered ($F_g = 0$).

Equation (3.1) has turned out to be very similar in form to the relations obtained in the literature by processing experimental data (see Table 1). The main parameter in (3.1) is the aperture diameter to the power 2.5, which affects the flow rate of the dispersed material. Like the relations in [4-8], Eq. (3.1) reflects the weak influence that particle diameter has on the flow rate. As follows from (3.1) and experiments, that influence is more pronounced for large particle and aperture diameters. A law of variation of the flow rate as a function of the particle diameter, which is similar to (3.1), was established experimentally in [7, 8]. As d_s/d_0 decreases the dependence on the particle diameter degenerates and (3.1) takes on the form of that in [3].

The factor K in (3.1) is not empirical. It can be calculated if the dome porosity ε is known, but such information is not available. Moreover, we must note that the possible range of ε is fairly narrow and as a result even an arbitrary choice of ε in the possible range does not increase the error of determination of K by more than 10%.

In the derivation of Eqs. (2.20) and (2.22) it was assumed that particles on the surface of the dynamic dome have the least stable cubic packing. The porosity is $\varepsilon = 0.47$ for cubic packing in the static state and should be slightly higher in the mobile state. That increase cannot be evaluated within the framework of the model under consideration. Experimental data on the determination of the porosity of vibrationally fluidized and moving layers as well as on a layer in the state of pseudofluidization suggest that under these conditions the porosity is 5-10% higher than that of an immobile layer. Such an increase in porosity above 0.47 as a result of the mobility of the particles can be expected in the given case. When the above is taken into account we can take the value $\varepsilon = 0.5$ in (2.21) and the corresponding value $K = 0.45$ (in accordance with (2.21)).

As comparison shows, Eq. (3.1) agrees fairly well with the empirical expression [15] as to form and quantitatively. It is noteworthy that the average value of the proportionality factor, determined by the theoretical relation (2.21) for $\varepsilon = 0.5$, agrees to within 20% with the average value of that factor, found experimentally in [3, 7, 15] for free-flowing materials. Equation (3.1) which contains no new empirical constants, therefore, gives a fairly good description of experimental data on gravity flow of a dispersed material through a single aperture into free space. It must be noted that comparison of the experimental data of different workers reveals a discrepancy of more than 20%. This can be attributed to the influence of factors not usually included, such as an excessive moisture content of the material, electrostatic forces, influence of the aperture edges, and so forth.

Free Flow of a Dispersed Material in the Presence of a Gas Counterflow. A comparison of the calculated results from the relation obtained and published experimental results on gravity flow of a granular material through an aperture in the presence of a gas counterflow is illustrated in Figs. 2 and 3, where the lines represent data calculated from (2.20) with allowance for (2.18) and (2.21) and the points represent the experimental results. The porosity ε of particles in the dome was assumed to be 0.5, as in the case of free gravity flow. The experimental data in Fig. 2, which were borrowed from [5], correspond to the free flow of chamotte powder from an aperture with $d_a = 0.04$ m [1, 1') $d_s = 0.0015$ m, $\rho_d = 990$ kg/m³, 2, 2') $d_s = 0.0025$ m, $\rho_d = 970$ kg/m³, 3, 3') $d_s = 0.004$ m, $\rho_d = 920$ kg/m³, 4, 4') $d_s = 0.0065$ m, $\rho_d = 900$ kg/m³]; the data in Fig. 3, which were taken from [10], correspond to the flow of particles under the following conditions: a) aperture 0.0125×0.0125 m, $d_s = 0.625$ mm, $\rho_s = 2420$ kg/m³, b) aperture 0.0125×0.0125 m, $d_s = 1.025$ mm, $\rho_s = 2500$ kg/m³, c) aperture 0.0145×0.0145 m, $d_s = 1.425$ mm, $\rho_s = 2540$ kg/m³. The coordinates in Figs. 2 and 3 were taken to be the same as in [5, 10] ($Q = \pi d_a^2 W_g/4$ is the gas flow rate through the aperture, $j_v = j_m/\rho_d$).

It is seen from Fig. 2 that in the main range of gas velocities the calculated particle flow rates deviate from the experimental data by less than 30%. The relative deviation may be more significant at near-critical gas velocities as $j_m \rightarrow 0$ (e.g., data 2, 2'). When the instability of free flow noted in [10] for free flows at near-critical values is taken into account,

however, the agreement between the experimental and calculated results can be considered to be entirely satisfactory. Unfortunately, the critical velocity at which free flow ceased was not recorded in [5]. At the same time extrapolation of the experimental results to $j_m = 0$ gives W_{ac} values that are close to the calculated values.

Calculated and experimental results obtained in [10] for the velocity of free flow of a granular material from rectangular apertures are compared in Fig. 3. The calculations were done with the equivalent diameter determined from $d_{ae} = 2(S_a/\pi)^{1/2}$. As is seen from Fig. 3, qualitatively the proposed theory faithfully reflects the law of variation of the flow rate of a granular material as a function of the gas velocity through an aperture. The difference between the results calculated from (2.20) and (2.22) differ from the experimental data for the flow rate of particles through an aperture is less than 30-35% and for the critical gas velocity, 5-7%. Such agreement for the process under consideration can be considered to be entirely satisfactory.

From a comparison of the calculated and experimental results we can conclude that the developed elementary theory, which does not include new empirical constants, agrees well with experimental data and makes it possible to describe the velocity of free flow of a dispersed material from a single aperture in the presence of a gas counterflow over the entire range of velocities. The divergences can be attributed to the influence exerted on the process by uncontrolled factors: the moisture content of the dispersed material, electrostatic forces, vibration, polydispersity, etc. Taking these factors into account is a problem for the further development of the theory of the free flow of dispersed material from an aperture.

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